The Space Elevator Feasibility Condition

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Abstract

This paper ties together parameters pertaining to tether specific strength and to power system mass density to arrive at an inequality that determines whether a Space Elevator system is viable.

The principle for the feasibility condition (FC) is that a Space Elevator must be able to lift its own weight fast enough – fast enough to grow by bootstrapping, fast enough to replace aging material, and fast enough to have a significant margin for commercial cargo beyond these housekeeping tasks. The FC therefore sets a 3 dimensional design space comprised of {specific strength, power density, survival time span}.

After developing the condition, real life limitations on power density and specific strength are plugged in, and the resultant viable design subspace is examined. Finally, a design architecture that satisfies the Feasibility Condition is briefly introduced.

1 Motivation

As is well known, there is no hard minimum requirement on the specific strength of a Space Elevator tether. The lower the specific strength, the higher the taper ratio, and the heavier the tether gets for a given climber mass. Previous work (Edwards, [4]) cites a UTS of 130 GPa for CNTs, and a density of 1.3 g/cc, for a specific strength of 100 GPa-cc/g, and a taper ratio of 1:2. This number is only predictive of course, but the implication is that if CNT tethers will end up being weaker then that, the only effect will be an increase in the mass of the Space Elevator tether.

As is also well known, there is no absolute requirement on the power system of the Space Elevator. Higher power levels allow the climber to move faster and clear the bottom of the tether sooner, so the launch rate and mass throughput end up higher. In [4] Edwards cites a 2 MWatt power system weighing 5 tons (0.4 kWatt/kg), able to launch a climber about once a week, and again the implication is that if this power density cannot be reached, the only penalty will be a reduction in the possible payload mass throughput.

The underlying reasoning in both these cases is that these are just quantitative changes that might affect performance and the eventual rate of ROI, but won't change the fundamental qualities of the Space Elevator such as scalability, launch environment, safety, or above all, feasibility.

There is, however, another assumption in the Space Elevator architecture that ties these two parameters together. A Space Elevator is too heavy to launch directly, and so the only way to construct a viable-sized Space Elevator is to launch a smaller seed Space Elevator, and use its lifting capacity to bootstrap to a much larger Space Elevator. In addition, the tether material will have a certain expected lifetime in space, and so the entire tether mass has to be replaced at a certain minimum rate.

These "housekeeping" lift requirements do not allow us to arbitrarily have a heavier tether and a slower launch rate. If the housekeeping chores cannot be kept, then this becomes an issue of feasibility, not performance. We therefore cannot have a simultaneously weaker-than-expected tether and weaker-than-expected power system – one clearly had to come at the expense of the other.

Since the Space Elevator is linearly scalable, we normalize the calculations by the maximum mass that is allowed to hang from the bottom of the tether (m_{max}). Thus a "20-ton" Elevator is one that can support a single 20 ton climber at ground level. Typically, this means that the tether weighs 4000 – 6000 tons, and the climbers will actually weigh around 15 tons (since we have multiple climbers simultaneously on the ribbon). Using m_{max} normalized mass units, the tether weighs 200-300, the climber weighs 0.75, etc. We define a "Standard throughput unit" (STU) as launching one m_{max} per year. STUs are typically applied to payload throughput.

To further simplify matters, we divide the mass of the climber into payload and power system, assuming the "dead structure" is small in comparison to either of them. (Any structure that scales with the power system (e.g. motors) is incorporated into the overall power density of the power system).

2 Feasibility Condition

The tether mass ratio (TMR) of the Space Elevator is the ratio of the tether mass and m_{max}. The tether specific loading (TSL) is similar to the tether specific strength, but takes into account parasitic mass (such as cross-weaves) and the margin of safety, so is the effective tether specific strength used in the design. Given a specific TSL, we can calculate the taper ratio and the TMR. (See “the tether equation”)

The payload mass throughput (PMT) of the Space Elevator is the amount of mass it can move per unit time, and is the product of the climber mass, the climber payload ratio and the launch rate. Given specific power system parameters such as power mass density and maximum allowed travel speed, we can calculate the PMT (See “Power system optimization”)

The Space Elevator Feasibility Condition (FC) is built around the concept of the characteristic time constant (CTC) – the time it takes the system to lift a payload mass equal to the mass of its tether. The CTC is simply the ratio of the tether mass and the payload mass throughput of the system: CTC = TMR/PMT.

When bootstrapping, the Space Elevator relies on a certain growth rate, or time-to-double (TD). During the initial growth phase, when starting from a seed Space Elevator that is only 5%-10% of the first operational Space Elevator, a TD of 1 year will result in a 4-year construction period in which the Elevator is not productive.

Since we want to hold a spare tether spool in orbit to enable reasonable recovery from a tether break event, we need to count on this additional mass being launched as well. We’ll denote the fraction weight of the spare-in-orbit as FS. If, for example, FS = 25%, then in order to double we actually need to launch 1.25 times the mass of the ribbon, and if TD is still 1 year, it will take 2 doublings and therefore 2 years to fully recover from the a broken tether.

While this example does not absolutely specify that TD has to be 1 year or that FS has to be 25%, it is clear that we cannot deviate too far from these values.

We call the periods where the Space Elevator has to grow rapidly, either during initial construction or during recovery from a break, “growth periods”, to be contrasted with “normal operations”.

During normal operations, the tether material will have some degradation rate in space as a result of cosmic radiation, micro orbital debris, thermal cycling, and perhaps simple mechanical wear and tear. These factors will result in an allowed material lifetime in service (TL). TL will be several times larger than TD, probably at least 4-5 years. We’ll assume that the spare does not degrade, and that $TL > 2\cdot TD$.

To support growth, the Space Elevator must lift (1+FS)/TD of its mass per year. In a similar manner, to replace aging material, the Space Elevator must lift 1/TL of its mass per year. We call these tasks “housekeeping lift requirements”.

Since during growth periods the housekeeping lift requirements are much higher than during periods of normal operations, it is enough to require that the Space Elevator will exactly support the housekeeping operations during growth periods, and use the additional capacity during normal operations for lifting payload. The sum (1+FS)/TD + 1/TL is therefore the minimal required lift capacity of the Space Elevator, in units of ribbon-mass-per-year.

The Feasibility Condition can thus be phrase as follows:

$$\frac{PMT}{TMR} = \frac{1}{CTC} > \frac{1+FS}{TD} + \frac{1}{TL}$$

The encouraging conclusion is that if a Space Elevator satisfies the Feasibility Condition, then during normal operations its minimum payload throughput is a very impressive $TMR\cdot(1+FS)/TD$ – easily 100 m_{max} per year when plugging in real-world numbers.

The discouraging news (as we’ll find out in the next section) is that satisfying the Feasibility Condition is rather difficult, so will impose strict conditions on some of the technologies and will rule out others.

3 Parameter values

Conservatively, we'll use TD = 1 years, FS = 25%, and TL = 4 years, so that the FC requires CTC < 0.66 years. As explained above, this number can be relaxed somewhat by using a smaller spare, agreeing to a longer TD, etc, so we can potentially relax the condition to TD = 1.5 years, FS = 25%, and TL = 6 years, so the FC becomes CTC < 1.0.

	TSL	A/A_0	TMR	P_{REQ}		P_{POS}	PD	
	Tether specific loading	Taper ratio	Tether mass ratio	Required mass throughput		Possible mass throughput	Power density	
	MYuri			STU		STU	kWatt/kg	
(optimistic)	50	2.6	50	50 – 75	<	100 – 115	0.5	(pessimistic)
	40	3.4	77	77 – 116		135 – 155	0.7	
CNT	30	5.0	144	144 – 216		170 – 210	1.0	Thin-film PV + motors
	25	7.0	228	228 – 342		200 – 275	1.5	
	20	11.3	433	433 – 650		230 – 340	2.5	
(pessimistic)	17	17.30	739	739 –1109		250 – 370	3.5	(optimistic)

Table 1: Tether-derived vs. Power-derived constraints [10],[11]

Choose two colors: Table 1 above shows the constraints imposed by the FC, and possible technology values (in gray) as detailed in the next section. Blue values are best, red are worst, P_{POS} must be greater than P_{REQ}. As expected, strong tethers match weaker power systems and vice versa. The gray zones are the probable performance levels of the technologies.

As it turns out, realistic CNT tether performance levels require very powerful climbers, up to 10 MWatt for a 20-ton Space Elevator) To fit such a power system into the climber requires a very power-dense system, and as we'll see below, this is not an easy requirement to satisfy.

4 CNTs and Power Beaming System Performance

Based on a gradual convergence of experimental and theoretical results, the specific strength of raw CNTs will not exceed 50 MYuri [6],[7], as compared to previous estimates of 100 MYuri[4]. In particular, a failure mechanism known as the Stone-Wales causes spontaneous defects in the Nanotube structure and limits the possible strength.

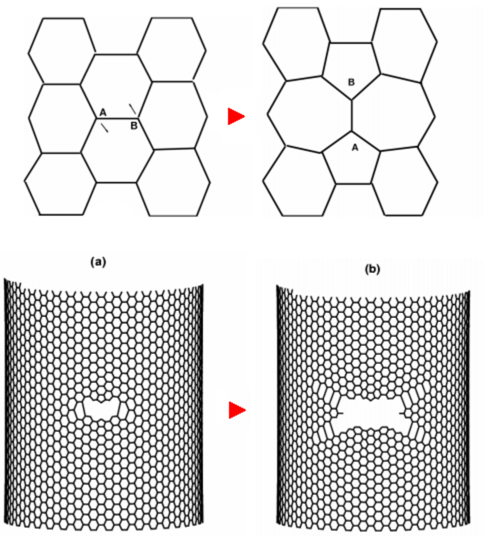

Figure 1: Stone-Wales Defect Formation [7]

Using 45-50 MYuri CNTs, we can expect a near-flawless spun tether to perform at 40 MYuri, and with a 33% safety margin, we can load the tether at a TSL of 30 MYuri. The weight of various redundancy structures can be shows to be only a few percent of the total tether mass, so will not affect this result by much.

Reaching a power mass density of 1.5 – 2.5 kWatt/kg is difficult. The best electric motors today achieve just under 1.5 kWatt/kg, leaving no margin for the PV panels. In order for the complete power system to reach 1.5 kWatt/kg, electric motor weight needs to be reduced by a factor of at least 2, though there's no theoretical barrier preventing this from happening. Ironically, CNT based conductors can provide a large leap in performance in that respect.

The power mass density of the PV system is mostly a function of the area required to radiate the excess heat from the panels. At 330 K (60 C), the rejection heat area density (at 100% emissivity) is 0.7 kWatt/m^2 per side, which is why a thin black plate at thermal equilibrium will reach a temperature only slightly above room temperature.

Following this rule of thumb, if the panel heat load is 10%, it can absorb 10 suns and still radiate the heat while operating at 60C. If the heat load is 30%, the power density needs to be dropped to 3.3 suns. If the heat load is 50%, the power density drops to 2 suns. Increasing the radiating area using external radiators adds a prohibitive amount of mass. [5]

It is important to remember that heat is created at the panels for many reasons. On top of the intrinsic inefficiencies of the cells, we need for to add (for example) effects of non-uniform illumination. A PV panel can operate efficiently only if all of its serially-connected elements produce the same amount of current. The problem is that a diffraction limited laser beam is Gaussian shaped, so is far from uniform. While theoretically a panel can be constructed to conform to that profile, any deviation in tracking, or any deviation from the nominal Gaussian profile, will immediately cause heating in the panel. The only way to reduce this effect is to use a non-diffraction limited beam, which requires a larger diameter spot, and so again argues for lower power densities.

As the power density drops, the size of the receiver increases, and together with the much higher power requirement imposed by Table 1, the size of the receiver reaches hundreds of meters in diameter for a 20 ton climber, and since the structure has to operate in 0 to 1 g, this becomes very difficult.

Since the ideal power density is now much lower, perhaps only 2-3 suns, it becomes interesting to consider direct solar illumination. Solar light is uniform, all pervasive, and free. The PV panel can be broken into a vertical chain of smaller panels that will not shade each other (unlike the laser, solar light is rarely along the direction of the tether)

To get a sense of what a system having the required power density of 1.5 kWatt/kg looks like, consider the 400 m^2 thin-film PV receiver shown below. The panel weighs only 32 kg, including the booms and deployment mechanism. (It stows in the little suitcase at the center – note the people in the back for scale). The panel can provide 50 kWatt of electric power under A0 illumination, or 1.5 kWatt/kg.

Figure 2: Thin Film for-space PV Panel

Even this panel can barely satisfy the power density requirements we derived. This technology is currently converting sunlight at slightly under 10%, and under optimistic assumptions, the technology can eventually reach 3-5 kWatt/kg. This extra power will be needed due to maximum speed-of-travel consideration.

5 Conclusion

On the bright side, we were able to show that we can make up for weaker-than predicted CNT tethers by stronger-than-predicted power systems. On the down side, we saw that the Space Elevator Feasibility Condition is actually very difficult to satisfy.

On the bright side, direct solar power conversion using thin-film foils seems to be able to achieve the necessary power-mass density. On the down side, such structures are very frail, and especially considering that the structure must work in both normal gravity and zero-g.

On the bright side, given the realization that we need much more power than previously thought (up to 10 MWatt per climber for a 20 ton Space Elevator), the move to free solar power relieves us of the need to construct much larger power beaming stations.

A solar-power based Space Elevator system is described in more detail in “Solar Augmented Space Elevator System”.

6 References

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